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Teaching Mathematics
to Students with Dyscalculia and Mathematics Anxiety

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Research on math learning difficulties, ways of overcoming and support needed

Final report

Information about project

Erasmus+ School education strategic partnership project "**Teaching Mathematics to Students with Dyscalculia and Mathematics Anxiety**" offers concrete suggestions on how to work with students with dyscalculia and mathematics anxiety, focusing on supporting mathematics teachers to help them overcome students' learning difficulties in mathematics.

To promote math achievements in math in the project partner countries, **general objectives of the project**: to increase the level of achievements in math for students with dyscalculia and mathematics anxiety and strengthen the profile of the math teachers, by sharing knowledge, exchange experience and developing new educational products to contribute to the issue.

The project is implemented by different organizations (schools, NGO, municipal institution) from six countries: Latvia, Lithuania, Macedonia, Romania, Spain, Turkey.

Contents

Introduction

Mathematics is one of the logical and practical academic disciplines to be acquired as a key competence in the context of lifelong learning. The importance of mathematics in daily and professional life as well as the fact that level of the mathematical competence influences the quality standards of individual and social life has been proved by several studies.

Nevertheless, learners' knowledge of mathematics is deteriorating year by year and mathematics is one of the reasons for dropping out at all levels of education. The results often are associated with the laziness, lack of motivation or other circumstances of the learner.

However, if a student has not learned something, has not done homework, is not studying, it does not always indicate laziness or unwillingness to do it. In many cases, this indicates that the student is having difficulty learning math and may not be able to learn mathematics. The student may have learning difficulties associated with math disorders such as **dyscalculia** or **mathematical anxiety**.

Dyscalculia is a specific learning disability in math which can cause different types of math difficulties. Students with dyscalculia may have difficulty to understanding number-related concepts or using symbols or functions needed for achievement in math. They may have trouble learning to count and recalling math facts. They may also have poor number sense and not understand math concepts like “greater than” and “less than.” And they may struggle with remembering phone numbers, or keeping track of scores when they’re playing sports. Sometimes, these challenges can make students with dyscalculia feel anxious about having to do math-related tasks. But dyscalculia isn’t the same as math anxiety.

Math anxiety can make students question their abilities in mathematics, even if they have strong skills. Students with math anxiety are so worried about the prospect of doing math that their fear and nervousness can lead to poor performance on math tests.

Some students may have both **math anxiety** and **dyscalculia**. Both dyscalculia and math anxiety can impact students’ capacity to do mathematics and they performance in math. However, it doesn’t affect them only at school. The challenges can also create difficulties in daily life. It also affects the achievements of math in the country as a whole.

Therefore, in order to promote progress in mathematics in individual countries and in Europe as a whole, various organizations (schools, NGOs, local authorities) from six countries: Latvia (LV), Lithuania (LT), Macedonia (MK), Romania (RO), Spain (ES), Turkey (TR) have initiated and are currently implementing the Erasmus+ School Education Strategic Partnership project “Teaching Mathematics to Students with Dyscalculia and Mathematics Anxiety”

Within the frame of this project, the research on math learning difficulties, ways of overcoming and support needed was carried out with the **aims** to **identify learning difficulties in mathematics** from the point of view of teachers and parents, to **identify existing experiences of overcoming** these learning difficulties in mathematics, and to **determine the support needed by teachers, parents and children**. Several steps have been taken to achieve these aims:

- A survey of both parents and teachers was conducted;
- Round tables were organized in all partner countries;
- Focused interviews with the target groups were carried out.

This report contains a summary of all research activities.

Understanding of the concepts of "dyscalculia" and "mathematical anxiety"

Several educational studies show that mathematics learning difficulties are determined by a number of **cognitive and emotional factors**. Scientists[1] have also shown that if a student has a math learning difficulty such as dyscalculia, he or she has calculation difficulties, impaired understanding of numerical concepts and mathematical activities, difficulties in understanding and working with numerical information without words, visually and spatially, often makes calculation errors and has difficulty performing mathematical activities. It should be noted that this is not a disease, but a **feature of the pupil's brain** that determines the perception and processing of information.

In turn, **mathematical anxiety** is one of the **emotional factors** that causes learning difficulties in mathematics. The term "mathematical anxiety" is described by scientists[2] as a fear of math or as a negative emotional reaction to mathematics which can be caused by a number of factors:

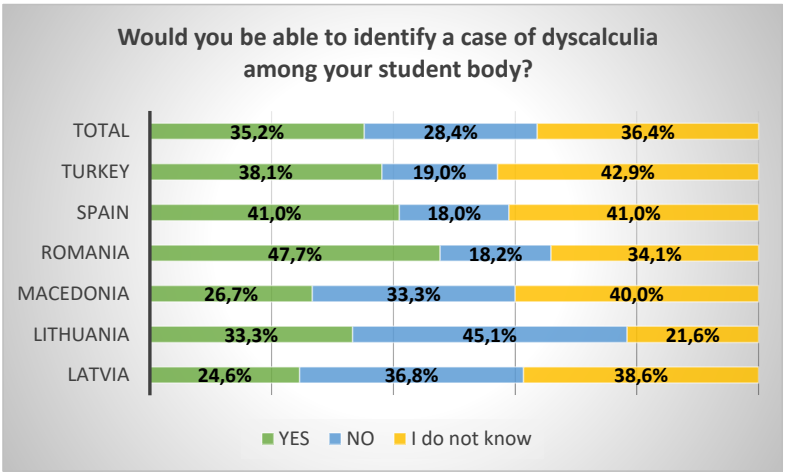
- lack of appropriate mathematical knowledge,
- learning strategies, application of mathematics in real life,
- limited exam time,
- lack of specific materials as well as personality type,
- lack of confidence, and also stereotypes related to teacher and
- parent approaches to mathematics.

Educational studies[3] show that students' mathematical achievement increases when anxiety is controlled, but dyscalculia lasts a lifetime. Of course, performance can improve if intensive developmental action is taken. The student needs to gain an understanding of how to use mathematics in everyday life; parents and teachers need to work together to identify the strategies needed. Scientists[4] suggest that the link between dyscalculia and/ or mathematical anxiety and performance can be driven.

In a **survey** conducted in all partner countries involved in the project, **math teachers** were asked if they would be able to identify a case of dyscalculia among their student body. As

[1] Rotzer, S., Loenneker, T., Kucian, K., Martin, E., Klaver, P., von Aster, M. (2009). Dysfunctional neural network of spatial working memory contributes to developmental dyscalculia. In Neuropsychologia 47 (2009) 2859-2865 Elsevier.
[2] Dowker, A., Sarkar, A., & Looi, C. Y. (2016). Mathematics anxiety: What have we learned in 60 years?. Frontiers in psychology, 7, 1-16. DOI: 10.3389/fpsyg.2016.00508.
[3] Kamann, M. P., & Wong, B. Y. (1993). Inducing adaptive coping self-statements in children with learning disabilities through self-instruction training. Journal of Learning Disabilities, 26(9), 630–638. DOI: 10.1177/002221949302600913.
[4] Turkington, C., Harris, J.R. (2006). The Encyclopedia of Learning Disabilities: Second Edition. United States of America: American Bookworks.

seen in the picture, only 35.2% of respondents answered in the affirmative, 28.4% answered no, but 36.4% - that they do not know.



A comparison of the responses by country shows that Romanian mathematics teachers are the most often are able to recognize dyscalculia. Almost half of the respondents (47.7%) answered in the affirmative.

The most critical in this respect are Latvian (24.6%) and also Macedonian (26.7%) teachers.

In view of this fact, **round tables** were organized in all partner countries with the aim to examine the results from the teachers and parents survey. In all countries the round table was composed by participants who are related to the teaching environment and who can participate actively in the activity: experts in education, teachers (mainly in mathematics), pedagogues and/or policy makers in the field of education.

Results from the math teachers survey: dyscalculia, knowledge and awareness - were put on the table. The following issues were raised for discussion:

- *How do you understand the concept of "**mathematical dyscalculia**" and "**mathematical anxiety**" and how does it differ from other types of learning difficulties (or laziness)?*
- *Briefly define the difference between 'mathematical dyscalculia" and "mathematical anxiety".*

Although the views of the roundtable participants and experts varied from country to country, they precisely define both dyscalculia and math anxiety. The results of the round tables by country are summarized below.

LV

- **Dyscalculia** is a specific ability to learn math (not an inability), a feature of the pupil's brain that determines the perception and processing of information.
- **Mathematical anxiety** - negative emotional reaction to math determined by different factors.
- Dyscalculia is a **cognitive disorder**, math anxiety – **emotional**.
- Math anxiety **can be controlled**, but dyscalculia **lasts a lifetime**.

MK

- **Dyscalculia** is a learning difficulty that affects math calculations. The severity of the condition varies from person to person.
- The main areas of weakness that signal the diagnosis of dyscalculia include **visual-spatial** difficulties and **language processing** difficulties.
- Other common symptoms include problems **learning math facts**, problems **recognizing numbers**, and problems **solving math problems**.
- Children who do not have adequate **math vocabulary** and **visual-spatial associations** may have difficulty mastering advanced mathematics.

RO

- The concept of "**dyscalculia**" and "**mathematical anxiety**" are not familiar for all teachers and educational experts, but the difficulties in learning math are well-known in the educational system.
- In general, teachers and other educational experts correctly identify **dyscalculia** as a learning deficit that only affects math, with different levels of intensity and present from early childhood.
- **Mathematical anxiety** is perceived either as fear of math operation in everyday life (e.g. counting money) or fear of assessment in math lessons.

ES

- **Dyscalculia** can develop from the earliest ages, offering problems when incorporating **basic logical-mathematical concepts, problem solving**, **psychomotor aspects** such as laterality or orientation, inversion of mathematical signs.
- It is based on **three main criteria**: specificity, discrepancy, exclusion.
- **Math anxiety** is not a learning disorder, but a lack of ability in learning math generated by emotional aspects that leads to confusion and problems in the execution of tasks and operations. It causes frustration, mental blocking, demotivation and fear of failure, and usually occurs in more advanced stages.
- As a main difference, **dyscalculia** is listed in learning difficulties tests as a "**math calculation disorder**" and can be determined in **a psycho-pedagogical report**. **Anxiety** is an **emotional effect** that causes mental blocking, mistrust, discomfort with the subject, fear, annulment or overwhelm that leads students to make mistakes and have problems with math even if they do not show any learning difficulties and are even a brilliant student.

TR

- **Dyscalculia** is a learning difficulty that affects math skills. Dyscalculia is not only a problem that affects student's school life at maths classes but also a **lifelong problem**.
- These children have trouble **remembering numbers** such as phone numbers, postcodes or game scores. **They struggle with money** matters, have difficulty judging the **length of distance**, struggle to remember **directions**, struggle to telling **time**. The impact of dyscalculia does not end when school ends, its affect **goes on after school**.
- Dyscalculia is congenital problem that is because of a disorder in **brain anatomy**. This is not an illness and it has no cure.
- Mathematical anxiety is an **emotional issue**. These students believe that they cannot achieve maths so this feeling creates failure. If it is determined on time, it can be healed by psycological support.

Types of dyscalculia

Several scientists[5] have studied and identified various signs of dyscalculia. In the case of dyscalculia, the symptoms can be very different:

- very poor mathematical ability,
- problems in using money, fear of money and transactions with it,
- unable to fully understand the meaning of numbers and evaluate numerical values,
- problems with such mathematical activities as addition, subtraction, multiplication
- problems with concepts such as sequence,
- poor understanding of directions,
- inability to read a map,
- understanding time, time schedule, observance of time,
- sequence of past and future events.

The survey conducted within the framework of this research was based on types of dyscalculia, which were identified by researcher L.Kosc[6], namely:

- **Verbal manifestation** of difficulties in verbally denoting mathematical concepts;
- **Practically Gnostic**, in which the systems of counting (calculating) specific and apparent objects or their symbols are disturbed;
- **Dyslexic**, which is based on mathematical sign reading disorders;
- **Graphic**, which manifests itself as a malfunction of mathematical notation and correct representation of geometric figures**;**
- **Operational**, associated with inability to perform mathematical operations.

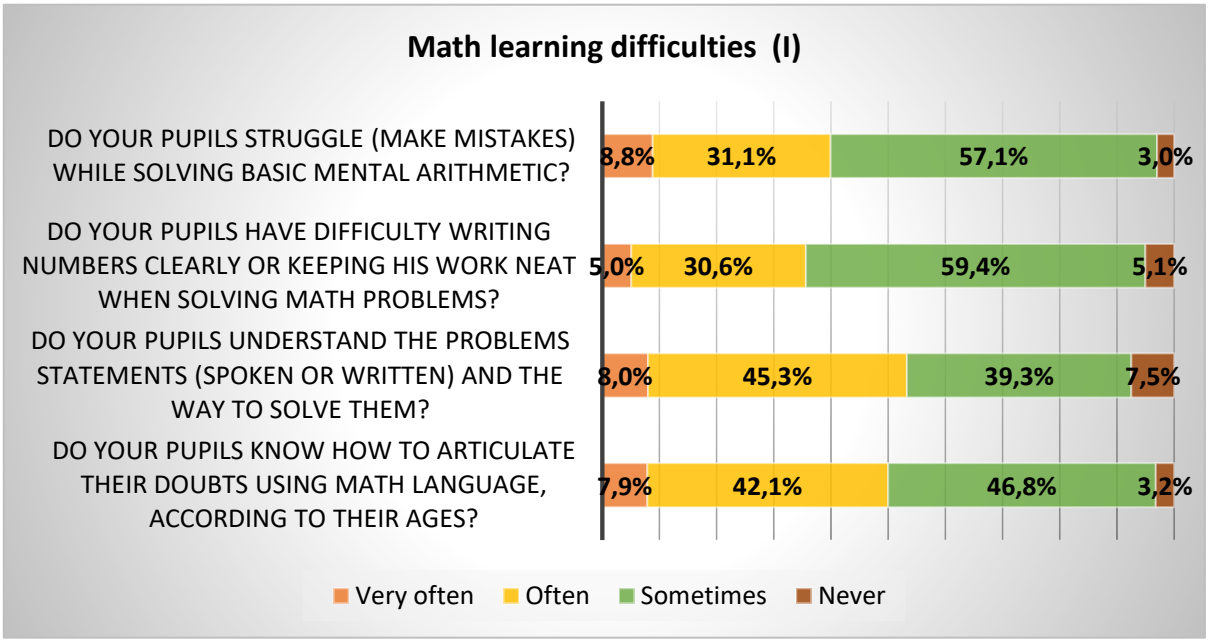
Types of dyscalculia	*Statements in the questionnaire*

[5] Turkington, C., Harris, J.R. (2006). The Encyclopedia of Learning Disabilities: Second Edition. United States of America: American Bookworks
[6] Kosc L. (1974) Development of dyscalculia. *Journal of learning disabilities*, 7 (3), 164-177.

Verbal manifestation	• Do your pupils know how to articulate their doubts using mathematical language, according to their ages? • Understand and verbalize mathematical concepts
Practically Gnostic	• Solve arithmetical mental calculations. • Solve mathematical Problems
Dyslexic	• Do your pupils understand the problems statements (spoken or written) and the way to solve them? • Struggle to identify mathematical signs like + and – and to use them the right way or has trouble understanding number symbols, like making the connection between "7" and the word "seven"
Graphic	• Do your pupils have difficulty writing numbers clearly or keeping his work neat when solving math problems? • Transcript mathematical signs.
Operational	• Do your pupils struggle (make mistakes) while solving basic mental arithmetic? • Struggle when comparing sizes or quantities.

Math teachers survey results

In the pictures below, summarised **math teachers'** answers to the statements on types of dyscalculia.



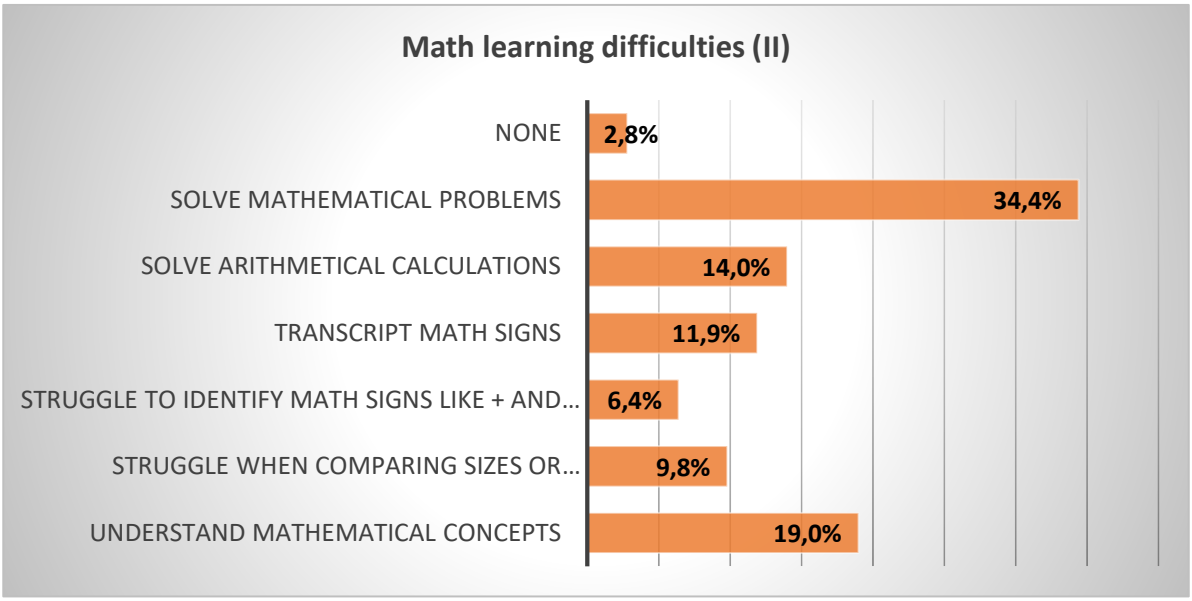
As seen in the graph, the biggest problem a **verbal manifestation** as a half of teachers who participated in the survey said that only half of their pupils (very often – 7.9%, often – 42.1%) know how to articulate their doubts using math language, according to their ages.

Almost half of the pupils show signs of **dyslexia**. This is evidenced by the fact that only 46.5% of pupils sometimes or never understand the problems statements (spoken or written) and the way to solve them.

The third types of dyscalculia - **operational** as about 40% of pupils struggle (make mistakes) while solving basic mental arithmetic.

35% of pupils have difficulty writing numbers clearly or keeping his work neat when solving math problems, which indicates the type of **graphic** type of dyscalculia.

To identify the types of dyscalculia teachers were asked about the main **Math learning difficulties** among their student body. There were several answers to this question, which describe different types of dyscalculia, and teachers had to point out which of the mentioned difficulties their students have. The results of the survey are summarized in the graph below.



Difficulties in solving mathematical problems, which indicate **practically gnostic** type of dyscalculia, were most often (in 34.4% of cases) noted in all partner countries.

Understanding and verbalizing mathematical concepts is the second most common difficulty (19%) in learning mathematics, which indicates the presence of **verbal manifestation.**

Third type of dyscalculia identified is **practically gnostic** as in 14% of cases teachers pointed difficulty to solve arithmetical mental calculations.

Difficulty to transcript mathematical signs was mentioned in 11.9% of cases and this difficulty characterizes **graphical** type of dyscalculia.

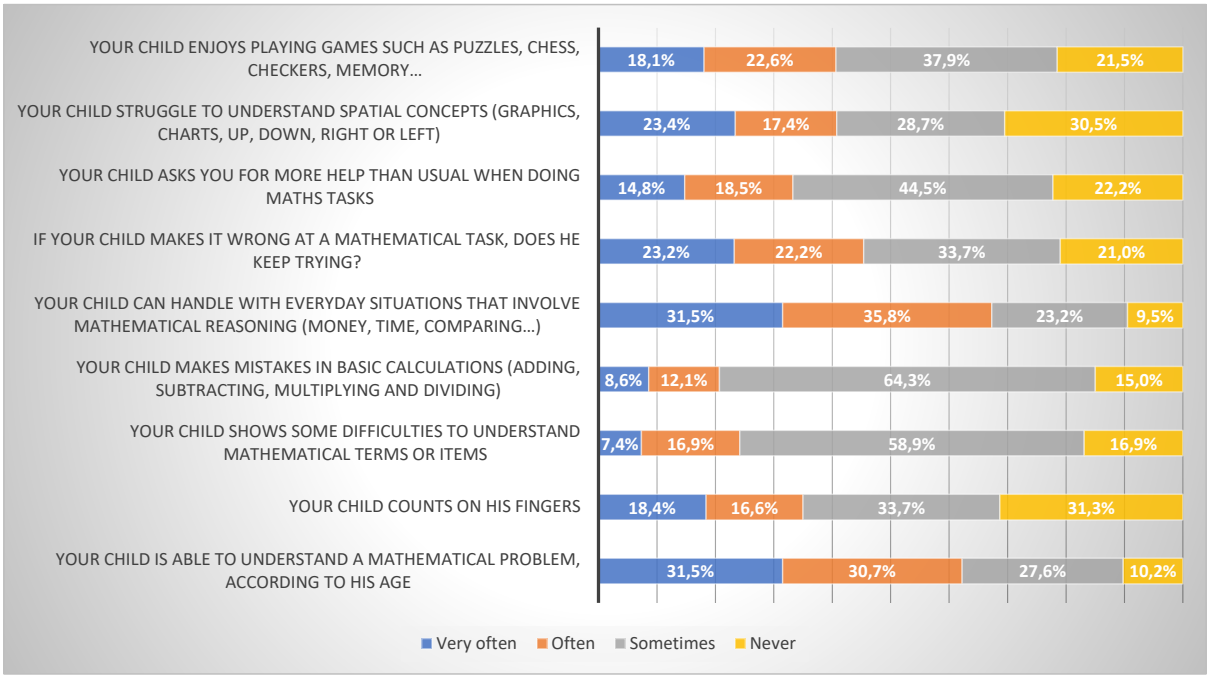
When performing the analysis of results between partner countries:

- Solving arithmetic calculations is the most problematic area in **Latvia**;
- Skills to articulate doubts using mathematical language, according to pupil ages as well as understanding the problems statements (spoken or written) and the way to solve them - in **Lithuania**;

- In **Macedonia** pupils more often struggle (make mistakes) while solving basic mental arithmetic and when comparing sizes or quantities;
- Struggle to identify math signs like + and – and to use them the right way or has trouble understanding number symbols, like making the connection between "7" and the word "seven" - is the most important math difficulty in **Romania**;
- **Spanish** pupils more often have difficulty to write numbers clearly or keeping his work neat when solving math problems as well as solve mathematical problems;
- **Turkish** pupils struggle (make mistakes) while solving basic mental arithmetic as well as solve mathematical problems.

Family survey results

The results of the study show that in all partner countries, parents are very optimistic about their children's learning difficulties in mathematics. Answers are collected in the graph below.



The vast majority of parents (67.3%) are convinced that their children **can handle with everyday situations** that involve mathematical reasoning (money, time, comparing…).

Although the experience of teachers shows that the **ability to understand a mathematical problem** according to their age is a problem in all partner countries, 62.2% of parents are sure that their children are able to understand a mathematical problem, according to his age and only 24% admit that their child shows some **difficulties to understand mathematical terms or items**.

In **Latvia**, according to parent survey results, only 4.4% of children **count on his fingers** and 82.6% of parents think that their children **can handle with everyday situations** that involve mathematical reasoning (money, time, comparing…).

In **Lithuania**, absolutely all parents think that their child is **able to understand** a mathematical problem, according to his age and only sometimes makes **mistakes in basic calculations** (adding, subtracting, multiplying and dividing).

According to parents, in **Macedonia** 84.6% of children **struggle to understand spatial concepts** (graphics, charts, up, down, right or left) and children more often (75.8%) enjoy playing games such as puzzles, chess, checkers, memory ….

In **Romania** 67.8% pf parents claim that their children show some **difficulties to understand mathematical terms or items** and 48.4% - that children make mistakes in basic calculations (adding, subtracting, multiplying and dividing).

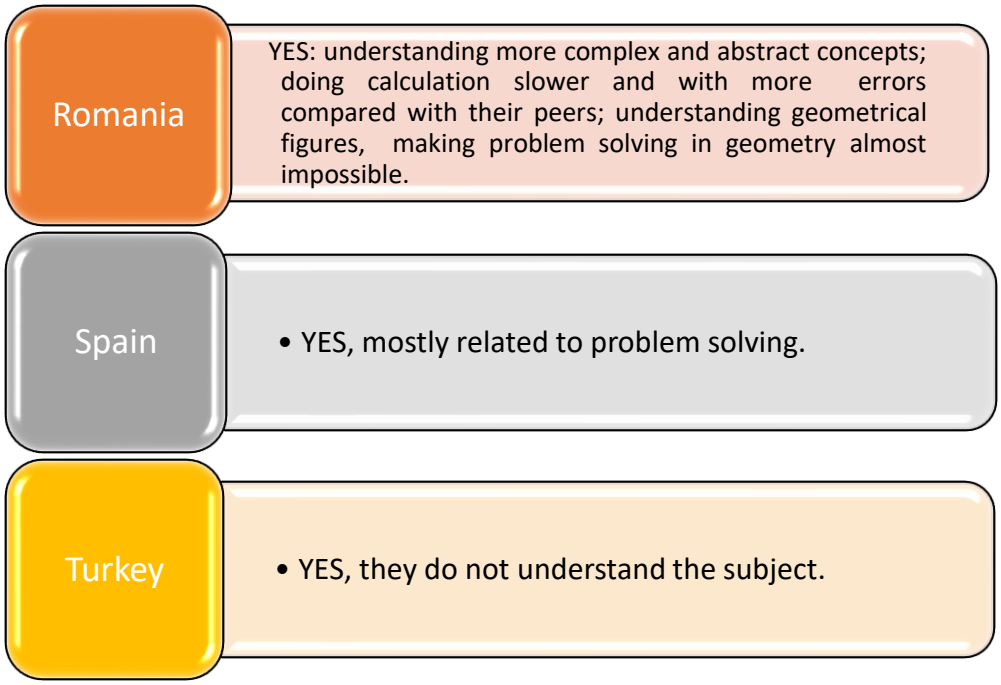
70.4% of parents in **Spain** admit that their children **count using fingers** and in 57.6% of cases very often or often they **use a calculator** for basic calculations at home.

In **Turkey** most often (in 71.4% of cases very often and often) child **asks parents for more help** than usual when doing Maths tasks, and only 42.8% of children **can handle with everyday situations** that involve mathematical reasoning (money, time, comparing…).

Results of student interviews

The project also conducted interviews with target groups, including pupils. They were asked: *"Are there students in your class who have difficulty with math? If yes, please describe what kind of difficulties are there?".* Answers given in the table.

Latvia	• Yes, difficulty to solve problems, to understand the new content, choose the right formula or task solving algorithm, others lag behind the pace of the class.
Lithuania	• YES. They are slower than average while understanding the subject, they simply need more time. There are students who are either lazy, or afraid, or just do not like maths.
Macedonia	• Difficulty mastering basic arithmetic operations, addition and subtraction. Remembering the basic mathematical procedures is also a problem.



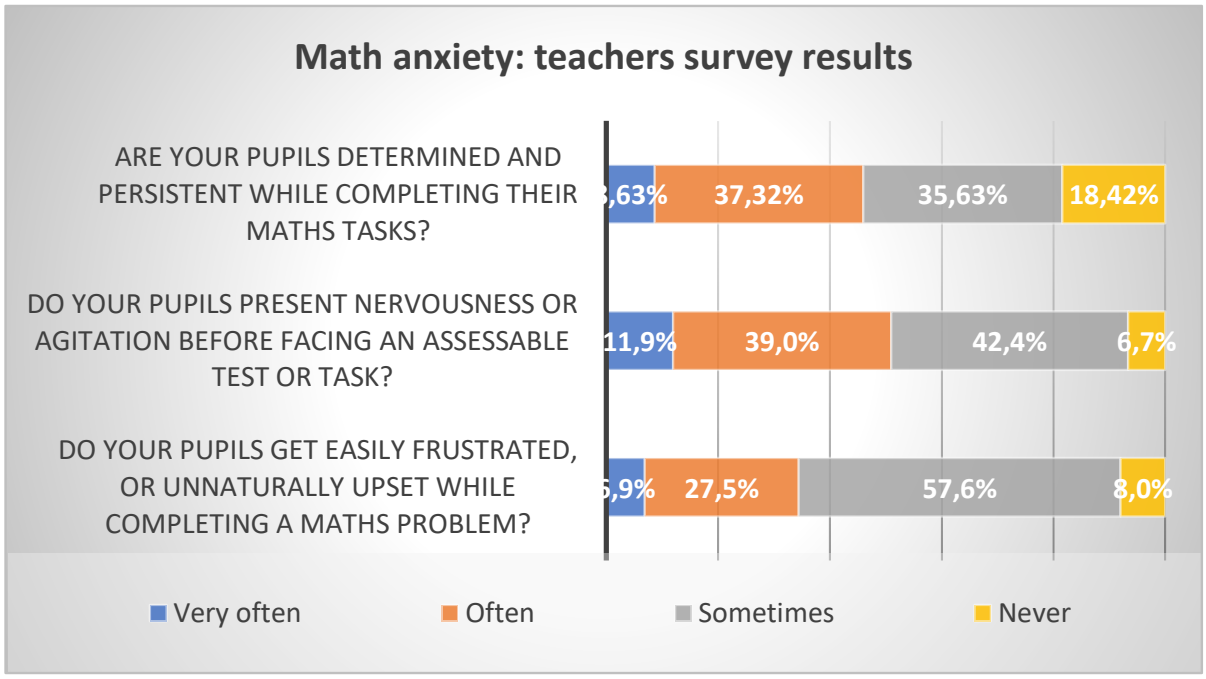
Mathematics anxiety

Math anxiety is an emotional effect that causes mental blocking, mistrust, discomfort with the subject, fear, annulment or overwhelm that leads students to make mistakes and have problems with mathematics even if they do not show any learning difficulties and are even a brilliant student.

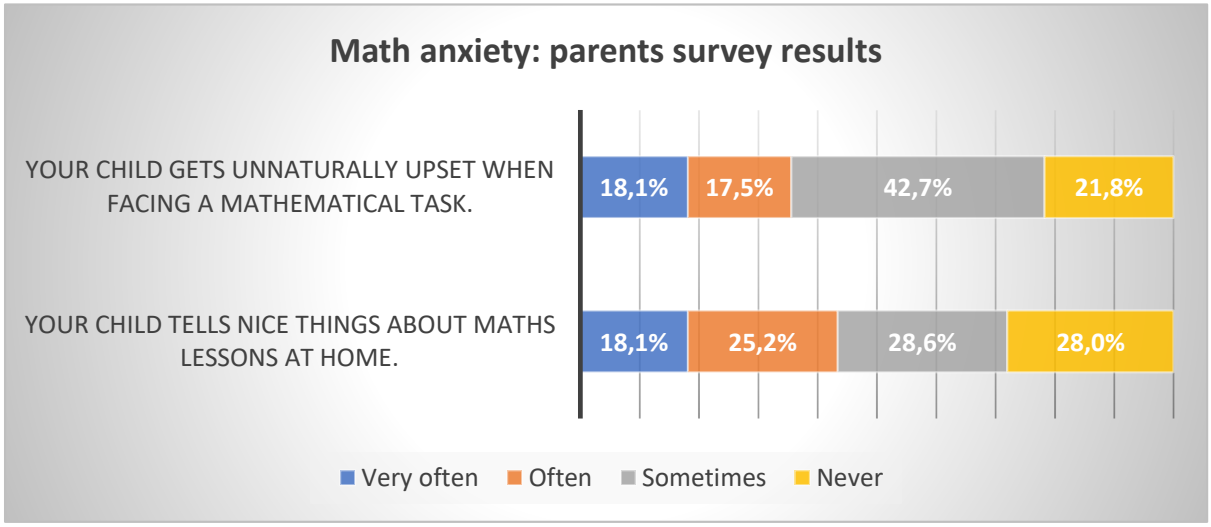
In order to identify math anxiety, both math teachers and parents were asked some questions (see table below) that the partners felt were indicative of math anxiety.

Surveys	*Statements in the questionnaire*
Math teachers survey	• Do your pupils get easily frustrated, or unnaturally upset while completing a maths problem? • Do your pupils present nervousness or agitation before facing an assessable test or task? • Are your pupils determined and persistent while completing their Maths tasks?
Family survey	• Your child tells nice things about Maths lessons at home. • Your child gets unnaturally upset when facing a mathematical task.

According to math teachers survey results a half of pupils (50.9%) very often or often **present nervousness or agitation** before facing an assessable test or task, but 3of pupils 4.4% get easily frustrated, or unnaturally upset while completing a math problem. 46% of pupils **are determined and persistent** while completing their Math tasks.



According to family survey results, only 43.4% of parents pointed out that their children very often or often **tell nice things about Maths lessons** at home. But 35.6% of parents have noticed that their children very often or often **get unnaturally upset** when facing a mathematical task.



An analysis of the results of the survey of math teachers by country shows that, in general, the highest level of math anxiety is in **Macedonia**.

Both **Latvian** and **Turkish** math teachers have observed that pupils very often or often present nervousness or agitation before facing an assessable test or task.

In **Spain** pupils most often are determined and persistent while completing their Maths tasks.

Analysis of the current situation regarding dyscalculia and mathematical anxiety

Behaviour of students with learning difficulties in mathematics

Both teachers and students were interviewed about the behaviour of pupils with math learning difficulties in class. The teachers were asked a question: *"How do the students who have math learning difficulty behave during maths lessons?"* The answers are collected in the table below.

Country	*Teachers answers*
	They are often **agitated**, **avoid** the gaze or other **contact** of the teacher, often engaged in side matters, sometimes **behave bravely and disturb others**; in the upper grades, they miss math classes or do not attend tests.
	Some of them are **silent, calm, reserved** and **are afraid of asking** questions. The others would rather keep on asking until they get answers. Some students even **do not make efforts to understand** the topic they deal with. They are mostly **passive**, **distracted** and **diffident** during the lessons.
	Frustration and tendency to establish **lower goals for future activities**, **avoidance strategies** *(including some older child skipping math classes when evaluation was announced),* potential **conflict with parents** because of lower grades in math and low interest in **communicating with the math teacher.**
	**Low participation in classes** due to insecurity, mental blocking and lack of motivation before tasks requiring logical-mathematical reflection.
	**Uninterested** during lessons - have concentration difficulties – teacher have to work with them individually

The same question was asked also to pupils: "*How do the students who have math learning difficulty behave during maths lessons?*"

Country	*Pupils answers*
	Some **ask the teacher**, **others students** who understand, but some **show no interest** at all and do not follow the lesson.
	They have **difficulty to concentrate** because they constantly **feel fear** and **tension** as they expect to get a question from the teacher during the lesson. Being **not motivated** students **do not pay attention to teacher's** explanations in the classroom, they just **copy the tasks from classmates** without making efforts to understand a certain topic

	They **were ridiculed** by classmates, although many classmates **tried to help** them learn.
	They **feel shy** and **cannot express themselves** well. Sometimes they **lose their attention:** don't listen to lesson carefully, they pay attention to other things. Some of them ask teacher to **explain again**.

The results of the research show that in order to overcome the difficulties of learning mathematics and also math anxiety, the methods and strategies to be developed within the project should also include methods or approaches to attract and retain pupils' attention, encourage and motivate them to participate, train concentration abilities, etc.

The cause of learning difficulties in mathematics from the parents' point of view

During the interviews, parents were asked: "*As you know, math is a stumbling block for many students. What do you think is the reason for this and what could help children better understand math and overcome anxiety?*" Results from parents' interview in table below.

Country	Parent answers
	Teachers explain **too quickly, lack of time to understand and solve tasks.** Nothing is **explained individually** in the classroom. The child more often **does not understand the connections** and often does not understand **what the teacher wants from him**
	• Very **intensive learning pace**, too **many new topics** and too **little time to go deeper** into them. • **Consistent work**, **constant repetition** and **creative tasks** could help immensely. • From the very start, there should not be any "gaps" left in the minds of pupils/students. • There could be **more practice with play elements** at primary school level
	It is necessary **to reduce the number of participants in the classes** so that the teacher can pay attention to each child individually.
	Difficulties in learning math are due to: 1) the **lower potential** of some children, and 2) the teaching **methods and teachers' skills and ability to teach** for each child. Tendency to focus on **external factors** such as **teaching**.
	**To contextualise** the subject more towards the student's immediate environment and everyday life through **real situations** and **constant examples.**

	• The **prejudice** is the biggest problem. • If the students love maths, they have self-confidence and they can succeed. • Teachers can do **funny activities, play games** in maths class. And parents can also do **funny maths activities** at home.

As can be seen, all the parents 'answers indicate that the reason for their children's difficulties in mathematics or mathematical anxiety is mainly related to the organization of the mathematics teaching process and also to the teachers' pedagogical skills and teaching methods.

Parents were asked to express their views on the matter: "*If a student has math anxiety, how does it affect achievements in math?*". The answers are different (see table).

Country	Parent answers
	**Stereotypes in society** that math is something terrible. It causes anxiety and creates blockages in learning math.
	Mostly the brain of those pupils/students **gets "blocked"**, does not accept any information; force children **to avoid** such "**uncomfortable situations**" in the future.
	The way **students' work is evaluated** must be changed immediately. Children are already anxious enough that they fail to achieve like their classmates, and sometimes even their **hardest work does not bring** a credited grade.
	**The anxiety** some students feel, parents consider it to be somehow **normal**, as grades are very important and, in many ways, decisive for the educational future
	Anxiety in mathematics **can lead to a drop** in their children's
	if a student has maths anxiety, he/she is afraid of maths and **this affects** his/her success.

As we know, the key to solving any problem is cooperation. Cooperation between parents and schools is also crucial in achieving better results in education. Therefore, parents were asked: "*Do you work with the school to solve your child's difficulties in learning math?*".

Parents in **Lithuania**, **Spain** and **Turkey** answered unequivocally "yes". **Latvians** also answered in the affirmative, but pointed out that disadvantaged families rarely do so. **Macedonians** think that it's something that does not always work.

Organization of the learning process for children with dyscalculia or math anxiety

The roundtable participants were asked how it is found that a pupil / student has mathematical dyscalculia or anxiety. The pooled results show that most participants were unaware of the existence of any dyscalculia identification procedures and specific interventions. Dyscalculia is usually formally identified by a special Commission as one of the learning difficulties in several countries:

	The **Pedagogical Medical Commission** decides on the degree of learning disabilities and the acquisition of a special education program in the case of severe general learning disabilities.
	The official identification of dyscalculia can only be made by the specialists from **Service for School and Professional Evaluation and Guidance** only at parents' requests.
	The process to determine whether a pupil has dyscalculia begins with observation by the teacher or tutor; it goes through the **Guidance Department** and a **psycho-pedagogical assessment** to determine whether there are special educational needs.
	When the teacher observes a student and thinks that this student has a kind of problem, first the **counsellor at school** talks to the student and parents. Then, they are directed to **psychiatrist**.

In a regular classroom, teacher can observe and advise parents to turn to this Commission and perform testing. Parents need to understand what this would do for the child, such as longer time for tests, slower pace of learning, individualized approach, additional counselling, etc.

To determine dyscalculia, observation of parents is very important but they generally do not want to accept a disorder and they ignore it (**Turkey**). Parental consent is also required for a student to visit a specialist, such as a psychologist, etc. However, parents often hide that the child has learning difficulties, do not acknowledge them and do not even engage in dialogue with the teacher (**Latvia**). Therefore, teachers must be very careful in determining such students.